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A DIFFUSION MODEL FOR OPTIMAL DIVIDEND DISTRIBUTION FOR A COMPANY WITH CONSTRAINTS ON RISK CONTROL*

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Abstract. This paper investigates a model of a corporation which faces constant liability payments and which can choose a production/business policy from an available set of control policies with different expected profits and risks. The objective is to find a business policy and a dividend distribution scheme so as to maximize the expected present value of the total dividend distributions. The main feature of this paper is that there are constraints on business activities such as inability to completely eliminate risk (even at the expense of reducing the potential profit to zero) or when such a risk cannot exceed a certain level. The case in which there is no restriction on the dividend pay-out rates is dealt with. This gives rise to a mixed regular-singular stochastic control problem. First the value function is analyzed in great detail and in particular is shown to be a viscosity solution of the corresponding Hamilton-Jacobi-Bellman (HJB) equation. Based on this it is further proved that the value function must be twice continuously differentiable. Then a delicate analysis is carried out on the HJB equation, leading to an explicit expression of the value function as well as the optimal policies.

Key words, diffusion model, dividend distribution, risk control, optimal stochastic control, HJB equation, viscosity solution, Skorohod problem

AMS subject classifications. 91B70, 93E20 **PII.** S0363012900382667

1. Introduction. Recently there has been an upsurge of interest in diffusion models for optimal dividend optimization and/or risk control techniques (see Jeanblanc-Pique and Shiryaev [11], Asmussen and Taksar [2], Radner and Shepp [16], Boyle, Elliott, and Yang [3], H0jgaard and Taksar [8], [9], [10], Paulsen and Gjessing [13], and Taksar and Zhou [18]). In those models the liquid assets of the company are modeled by a Brownian motion with constant drift and diffusion coefficients. The drift term corresponds to the expected (potential) profit per unit time, while the diffusion term is interpreted as risk. The larger the diffusion coefficient the greater the business risk the company takes on. If the company wants to decrease the risk from its business activities, it also faces a decrease in its potential profit. In other words, different business activities in this model correspond to changing *simultaneously* the drift and the diffusion coefficients of the underlying process. This sets a scene for an optimal stochastic control model where the controls affect not only the drift but also the diffusion part of the dynamic of the system.

Another important feature of our paper is dividend distribution. Dividends are paid from the liquid reserve of the company and distributed to the shareholders.

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In **conclusion,** we would like to point out an intricate interplay between the liability and restrictions on the risk control of

a financial company. The sheer number of qualitatively different optimal policies, which appears due to different possible relationships between exogenous parameters, shows the multiplicity of different economic environments which a financial company faces depending on the size of the debt and on the size of available business activity.

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REFERENCES

[1] S. Asmussen, B. H0JGAARD, and M. Taksar (2000), *Optimal risk control and dividend distribution policies. Example of e.xcess-of loss reinsurance,* Finance Stoch., 4, pp. 299-324. [2] S. ASMUSSEN and M. Taksar (1997), *Controlled diffusion models for optimal dividend pay-out,*
Insurance Math. Econom., 20, pp. 1-15. [3] P. Boyle, R. J. Elliott, and H. Yang (1998), *Controlled Diffusion Models of an Insurance*
Company, preprint, Department of Statistics, The University of Hong Kong. [4] T. Choulli, M. Taksar, and X. Y. Zhou (2001), *Excess-of-loss reinsurance for a company with debt liability and constraints on risk reduction,* Quantitative Finance, 1, pp. 573—596. [5] C. Dellacherie and P. A. Meyer (1980), *Probabilite et potentiels: Theorie des martingales,*

[6]Hermann, Paris. [6] W. H. FLEMING AND R. W. Rishel (1975), *Deterministic and Stochastic Optimal Control,*Springer-Verlag, Berlin—New York.

[7] W. H. Fleming and H. M. Soner (1993), *Controlled Markov Processes and Viscosity Solutions,* Springer—Verlag, New York. [8] B. H0JGAARD AND M. Taksar (1998a), *Optimal proportional reinsurance policies for diffusion models with transaction costs,* Insurance Math. Econom., 22, pp. 41-51. [9] B. H03GAARD AND M. Taksar (1998b), *Optimal proportional reinsurance policies for diffusionmodels,* Scand. Actuar. J., 2, pp. 166—168. [10] B. H0JGAARD and M. Taksar (1999), *Controlling risk exposure and dividends pay-out schemes: Insurance company example,* Math. Finance, 2, pp. 153—182. [11] M. Jeanblanc-Picque AND A. N. Shiryaev (1995), *Optimization of the flow of dividends,* Russian Math. Surveys, 50, pp. 257-277.

[12] P.-L. Lions and A.-S. Sznitman (1984), *Stochastic differential equations with reflecting boundary conditions,* Comm. Pure Appl. Math., 37, pp. 511-537. [13] J. Paulsen and H. K. Gjessing (1997), *Optimal choice of dividend barriers for a risk process*
with stochastic return on investments, Insurance Math. Econom., 20, pp. 215—223. [14] D. REVUZ and M. Yor (1999), *Continuous Martingales and Brownian Motion,* 3rd ed., Springer-Verlag, Berlin.

[15] H. L. Royden (1988), *Real Analysis,* 3rd ed., Prentice-Hall, Upper Saddle River, NJ. [16] R. Radner AND L. Shepp (1996), *Risk vs. profit potential: A model for corporate strategy, i.*Econ. Dynam. Control, 20, pp. 1373-1393. [17] M. Taksar (2000), *Optimal risk and dividend distribution control models for an insurance*
company, Math. Methods Oper. Res., 51, pp. 1—42. [18] M. Taksar and X. Y. Zhou (1998), *Optimal risk and dividend control for a company with a debt liability,* Insurance Math. Econom., 22, pp. 105-122.

19] J. YONG and X. Y. Zhou (1999), *Stochastic Controls: Hamiltonian Systems and HJB Equations,* Springer— Verlag, New York.